TEMPERATURE FLUCTUATION BALANCE IN AN AXISYMMETRIC TURBULENT WAKE

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UDC 532.517.4:536.242

Results of experiments conducted in the thermal wake behind an axisymmetric body are used to evaluate the radial profiles of different terms of the balance equation for the mean square of temperature fluctuations.

Results were presented in [1] from measurements of the mean and fluctuation characteristics of turbulent velocity and temperature fields in the wake behind an ellipsoid of revolution. The completed experiment was used as a basis to conclude that "sensitivity" was greater to the initial conditions of the temperature field compared to the velocity field. Here we continue the study of the evolution of the temperature field in the wake of an ellipsoid by measuring "finer" characteristics — the cross-correlations of the velocity and temperature fluctuations — and we analyze the balance equation for the mean square of the temperature fluctuations. A similar analysis was made in [2], which studied the wake behind a heated sphere. In connection with this study, it is possible to evaluate the effect of the shape of a body on the behavior of different terms of this equation.

The balance equation for the temperature pulsations in the case of an axisymmetric wake can be written in the form

$$U\frac{\partial\overline{\theta^2}}{\partial x_1}+V\frac{\partial\overline{\theta^2}}{\partial x_2}+2\overline{u_2\theta}\,\frac{\partial\Delta T}{\partial x_2}+\frac{1}{x_2}\,\frac{\partial}{\partial x_2}x_2\overline{u_2\theta^2}+\varepsilon_\theta-\frac{\varkappa}{x_2}\,\frac{\partial}{\partial x_2}\left(x_2\,\frac{\partial\overline{\theta^2}}{\partial x_2}\right)=0. \tag{1}$$

The individual terms of this equation characterize the change in the fluctuations $\overline{\theta^2}$ as a result of convection, generation, turbulent diffusion, dissipation, and molecular diffusion. Each term of Eq. (1) was calculated numerically by selecting appropriate approximating functions describing the character of the change in experimental values of $\overline{\theta^2}$, ΔT, $\overline{u_2\theta}$, $\overline{u_2\theta^2}$ over the radius of the wake. The terms characterizing the transport of fluctuations $\overline{\theta^2}$ by convection and molecular diffusion were determined on the basis of the results in [1]. Turbulent diffusion and generation of the quantity $\overline{\theta^2}$ were calculated from measured radial distributions of the correlations $\overline{u_2\theta}$ and $\overline{u_2\theta^2}$. To obtain more complete information on the interrelationship of the velocity field and temperature field, we also measured the correlations of $\overline{u_1\theta}$, $\overline{u_1\theta^2}$, $\overline{u_1u_2\theta}$. The distributions of one-point mixed moments were obtained in two sections of the wake: $x_1/d = 30$ and $x_1/d = 50$. The method used to measure the cross-correlations of the velocity and temperature fluctuations in nonisothermal flows was described in [3].

Dimensionless values of the above quantities are shown in Fig. 1. It can be seen that similitude of the mixed-moment distributions is seen in the case of normalization for characteristic levels of fluctuations in the investigated section, while similitude is absent in the case of normalization for the means ΔU_0 and ΔT_0. This finding is consistent with the conclusion made in [1] regarding the nonsimilitude of the temperature field at the investigated distances from the model. The available measurements of correlations of velocity and temperature pulsations in the wake of an axisymmetric body [2] are limited to the moments $\overline{u_2\theta}$ and $\overline{u_2\theta^2}$. More detailed measurements of different correlations of this type have been made in the wake of a cylinder [4] and in a circular coflowing jet [5]. Unfortunately, the behavior of the measured correlations can be compared only qualitatively due to the different normalizations used in these studies.

The character of the change in $\overline{u_2\theta}$, $\overline{u_2\theta^2}$, $\overline{u_1\theta}$, $\overline{u_1\theta^2}$, $\overline{u_1u_2\theta}$ shown in Fig. 1 corresponds to the behavior of analogous quantities in [4, 5]. Some difference is seen for the correlation

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 181-187, February, 1986. Original article submitted November 19, 1984.

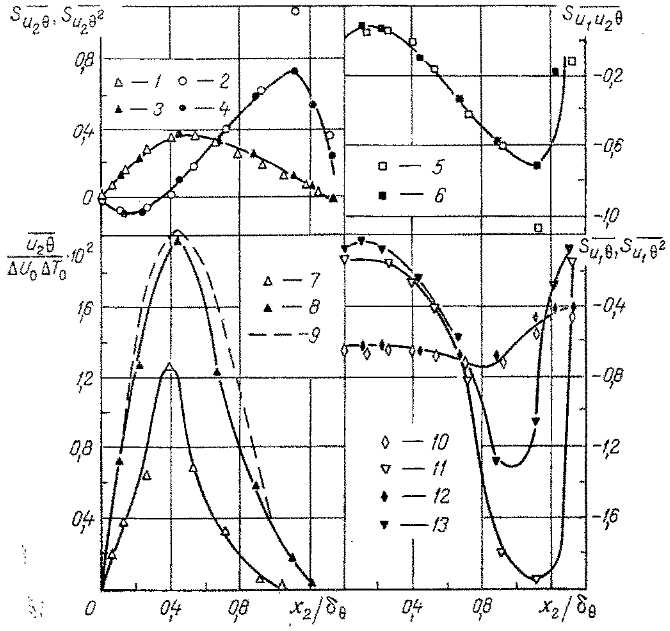

Fig. 1. Radial distribution of mixed moments of velocity and temperature fluctuations in a wake: 1) $S_{\overline{u_2\theta}}$, 2) $S_{\overline{u_2\theta^2}}$, $x_1/d = 30$; 3) $S_{\overline{u_2\theta}}$, 4) $S_{\overline{u_2\theta^2}}$, $x_1/d = 50$; 5) $S_{\overline{u_1u_2\theta}}$, $x_1/d = 30$; 6) $S_{\overline{u_1u_2\theta}}$, $x_1/d = 50$; 7) $x_1/d = 30$; 8) $x_1/d = 50$; 9) calculated values of $\overline{u_2\theta}$; 10) $S_{\overline{u_1\theta}}$, 11) $S_{\overline{u_1\theta^2}}$, $x_1/d = 30$; 12) $S_{\overline{u_1\theta}}$, 13) $S_{\overline{u_1\theta^2}}$; 13) $S_{\overline{u_1\theta^2}}$, $x_1/d = 50$.

of $\overline{u_1\theta^2}$, but this is connected with the fact that its self-similar profile is evidently established farther from the model than in the case of other mixed moments.

Comparison of the distributions of the moments $\overline{u_2\theta}$ and $\overline{u_2\theta^2}$ obtained in wakes of axisymmetric bodies of different shapes shows that there is a significant difference in the behavior of $\overline{u_2\theta^2}$. In our experiment, this correlation changes sign near the axis of the wake, while in the wake of a sphere [2] the quantity $\overline{u_2\theta^2}$ has a constant sign in this region. It should be kept in mind that the measurements in the wake of the sphere [2] were made in the self-similar flow region, while in our experiment the temperature field has not yet reached the self-similar state. In connection with this, it cannot be said with certainty that the observed differences in the behavior of $\overline{u_2\theta^2}$ are connected with the effect of the shape of the body.

The accuracy of measurements of the mixed moments was evaluated using the example of calculation of the correlation $\overline{u_2\theta}$. Here, we employed the results from [1]. The heat-transfer equation in the axisymmetric case is written in the form:

$$U\frac{\partial \Delta T}{\partial x_1} + V\frac{\partial \Delta T}{\partial x_2} = -\frac{1}{x_2}\frac{\partial}{\partial x_2}(x_2\overline{u_2\theta}) + \frac{\varkappa}{x_2}\frac{\partial}{\partial x_2}x_2\frac{\partial \Delta T}{\partial x_2}. \tag{2}$$

Integrating (2), we can determine $\overline{u_2\theta}$ from measured profiles of mean temperature by calculating the transverse velocity V with the continuity equation

$$\frac{\partial U}{\partial x_1} + \frac{V}{x_2} + \frac{\partial V}{\partial x_2} = 0. \tag{3}$$

A similar estimate was made in [2]. However, Eq. (2) was written in a long-range wake approximation. Our calculations for the section $x_1/d = 50$ showed that the contribution of the second term of Eq. (2) is quite large, so it must be considered. We represent the longitudinal velocity and the excess temperature in the form

$$U = U_1 - \Delta U_0 f(\eta), \tag{4}$$

$$\Delta T = \Delta T_0 g(\eta), \tag{5}$$

where ΔU_0 and ΔT_0 are the velocity and temperature defects; $f(\eta)$ and $g(\eta)$ are the corresponding dimensionless profiles and $\eta = x_2/\delta_\theta$ is the dimensionless transverse coordinate. Inserting (4) into (3) and integrating, we obtain a formula for the transverse velocity

$$-V = \Delta U_0 \left(\frac{d\delta_T}{\partial x_1}\right)\left[\eta f - 2\,\frac{1}{\eta}\,I_1\right] - \delta_T\left(\frac{d\Delta U_0}{dx_1}\right)\frac{1}{\eta}\,I_1, \tag{6}$$

where $I_1 = \int\limits_0^\eta f\eta d\eta$.

Solving Eq. (2) with allowance for (6), we find the theoretical relation for $\overline{u_2\theta}$:

$$-\frac{\overline{u_2\theta}}{\Delta U_0 \Delta T_0} = \left[\frac{\delta_T^*}{\Delta U_0^* \Delta T_0^*}\,\frac{d}{dx_1^*}(\Delta U_0^* \Delta T_0^*) + 2\,\frac{d\delta_T^*}{\partial x_1^*}\right]\frac{1}{\eta}\,I_{12} +$$

$$+\left(\frac{\delta_T^*}{\Delta U_0^* \Delta T_0^*}\,\frac{d\Delta T_0}{dx_1^*} + \frac{2}{\Delta U_0^*}\,\frac{d\delta_T^*}{dx_1^*}\right)\frac{1}{\eta}\,I_2 + \left[\frac{\delta_T^*}{\Delta U_0^*}\,\frac{d\Delta U_0^*}{dx_1^*} + 2\,\frac{d\delta_T^*}{dx_1^*}\right]\frac{1}{\eta}\,g I_1 - \frac{1}{\Delta U_0^*}\,\frac{d\delta_T^*}{dx_1^*}\,\eta g - \frac{1}{\mathrm{Re\,Pr}}\,g, \tag{7}$$

where $I_2 = \int\limits_0^\eta g\eta d\eta;\ I_{12} = \int\limits_0^\eta fg\eta d\eta;\ \delta_T^* = \delta_T/d;\ \ \Delta U_0^* = \Delta U_0/U_1;\ \ x_1^* = x_1/d;\ \Delta T_0^* = \Delta T_0/\Delta T_i$.

To determine the derivatives $d(\Delta T_0^*)/dx_1^*$, $d(\Delta U_0^*)/dx_1^*$, $d\delta_T^*/dx_1^*$, we used the empirical dependence of the quantities ΔT_0, ΔU_0, δ_T on the longitudinal coordinate:

$$\Delta T_0 = C_T (x_1^* - x_0^*)^{-2/3},\ \Delta U_0 = C_U (x_1^* - x_0^*)^{-2/3},\ \delta_T = C_\delta (x_1^* - x_0^*)^{1/3}, \tag{8}$$

where $x_0^* - x_0/d$ is the dimensionless virtual origin.

The dimensionless profiles of the velocity defect $f(\eta)$ and excess-temperature defect $g(\eta)$ were approximated by the functions

$$f(\eta) = \exp(-2{,}16\eta^2 - 1{,}353\eta^4 - 0.6\eta^6), \tag{9}$$

$$g(\eta) = \exp(-2{,}06\eta^2 + 0{,}335\eta^4 - 0{,}121\eta^6). \tag{10}$$

The constants in these formulas were found by the least squares method. The standard deviation of the relations from the experimental points was no greater than 2%.

The thus-calculated values of $\overline{u_2\theta}$ (curve 1 in Fig. 1) agree well with directly measured values, which is evidence of the reliability of the measurements of mean and fluctuation quantities in the present experiment.

Scalar dissipation is usually evaluated using the hypothesis of local isotropy of the temperature field, in accordance with which

$$\left(\frac{\overline{\partial\theta}}{\partial x_1}\right)^2 = \left(\frac{\overline{\partial\theta}}{\partial x_2}\right)^2 = \left(\frac{\overline{\partial\theta}}{\partial x_3}\right)^2, \tag{11}$$

from which

$$\varepsilon_\theta = 2\varkappa\left(\frac{\overline{\partial\theta}}{\partial x_i}\right)^2 = 6\varkappa\left(\frac{\overline{\partial\theta}}{\partial x_1}\right)^2. \tag{12}$$

To check the validity of (11), along with $\overline{(\partial\theta/\partial x_1)^2}$ we measured the quantities $\overline{(\partial\theta/\partial x_2)^2}$ and $\overline{(\partial\theta/\partial x_3)^2}$ with two "DISA" 55R31 temperature probes (filament diameter 1 μm, length 0.1 mm). The filaments of the probes were located parallel to each other. In the case of identical sensitivity of the probes, the difference between their signals is proportional to the space derivative of the corresponding coordinate (depending on the location of the filaments):

$$\frac{\overline{\partial\theta}}{\partial x_i} \sim \frac{\Delta\theta}{\Delta l} \sim \frac{e_1 - e_2}{\Delta l}, \quad (13)$$

where e_1 and e_2 are the signals of the temperature probes; $\Delta\ell$ is the distance between the filaments. The accuracy of Eq. (13) increases with a decrease in $\Delta\ell$. However, in this case there is also a decrease in the amplitude of the useful signal (and, thus a deterioration in the signal/noise ratio). Also, the probes begin to affect each other. Due to these factors, the minimum value of $\Delta\ell$ did not exceed 0.5 mm. The working current was about 0.8 mA, which allowed us to ignore sensitivity to velocity fluctuations.

Introducing the coefficients $K_{12} = \overline{(\partial\theta/\partial x_2)^2}/\overline{(\partial\theta/\partial x_1)^2}$ and $K_{13} = \overline{(\partial\theta/\partial x_3)^2}/\overline{(\partial\theta/\partial x_1)^2}$, we can represent Eq. (12) in the form

$$\varepsilon_\theta = 2\varkappa \left(\frac{\overline{\partial\theta}}{\partial x_1}\right)^2 (1 + K_{12} + K_{13}). \quad (14)$$

The quantities K_{12} and K_{13} were determined with an initial temperature difference between the jet and the incoming flow $\Delta T_i = 30°C$. The accuracy of the measurements of these quantities was low due to the low signal/noise ratio (it is equal to four at $x_1/d = 3$ on the wake axis and decreases to unity at $x_1/d = 20$). As a result, we could not evaluate the behavior of the coefficients on the edge of the wake. However, there is an obvious tendency for K_{12} to increase toward the external boundary and for K_{13} to be nearly constant (Fig. 2). The deviations of K_{12} and K_{13} from unit are not great (on the order of 20%), although they are greater than the corresponding values in the wake of a heated cylinder [6]. It should be noted that the equation $K_{12} = K_{13}$ should be satisfied on the wake axis due to the axial symmetry of the flow. It is evident from Fig. 3 that the experimental values differ by about 10%. The deviations of the temperature field from the locally isotropic case was ignored in the calculation of the dissipative term due to the low accuracy of the measurements of K_{12} and K_{13} and their slight difference from unity. More important were the corrections for the final resolution of the temperature probe [1], which reached approximately 100% in the section $x_1/d = 50$.

In calculating the convective and diffusive terms in balance equation (1), we approximated the radical profiles of the temperature fluctuations $\psi(\eta) = \overline{\theta^2}/\overline{\theta_0^2}$ and the triple mixed correlation $S(\eta) = \overline{u_2\theta^2}/\Delta U_0 \Delta T_0^2$ by the following empirical expressions:

$$\psi(\eta) = (1 + 7.32\eta^2)(1 + \eta^4 + 2\eta^6)\exp(-3.5\eta^2 - 0.4\eta^4 - \eta^6), \quad (15)$$

$$S(\eta) = 69\eta(\eta^2 - 0.084)\exp(-2.17\eta^4)[\exp(-8.5\eta^2) + 0.18]. \quad (16)$$

The terms of Eq. (1), made dimensionless in the complex $\delta_\theta/\Delta U_0 \Delta T_0^2$ (Fig. 3), show that the dissipative and convective terms contribute to the balance of $\overline{\theta^2}$ near the axis. Generation and dissipation predominate in the region of maximum temperature fluctuations ($0.3 < \eta < 0.6$), but the contribution of the remaining terms is also quite substantial (except for molecular diffusion, which is negligible for any value of η). Figure 3 shows two distributions of turbulent diffusion. One of them (curve 5) was calculated from empirical profiles of the mixed moment $\overline{u_2\theta^2}$ approximated by Eq. (16). The other (curve 6) was calculated as the closing term of balance equation (1). It turned out that the two distributions nearly coincide near the axis ($0 \le \eta \le 0.2$) and on the edge of the wake ($\eta > 0.9$), but the temperature differences between them is 40% of the largest balance term in the region of the temperature fluctuation maximum. The reason for this deviation is evidently the error from direct evaluation of the diffusion term from the profile $\overline{u_2\theta^2}$, which was caused by the small number of experimental points in the region of the abrupt increase in this moment. It is less likely for dissipation to be underestimated due to failure to allow for anisotropy of the temperature field.

In contrast to the balance of $\overline{\theta^2}$ obtained in the present study (Fig. 3), turbulent diffusion on the axis is close to zero and is positive in the wake of a sphere [2] (as in the case of an ellipsoid, the determining terms are dissipation and convection). There is an increase in generation in the region of maximum temperature fluctuations ($0.25 < \eta < 0.5$), but it is roughly 40% of the amount of convection — which is greater than all of the remaining terms of the balance equation (generation is the largest term in the case of an ellipsoid).

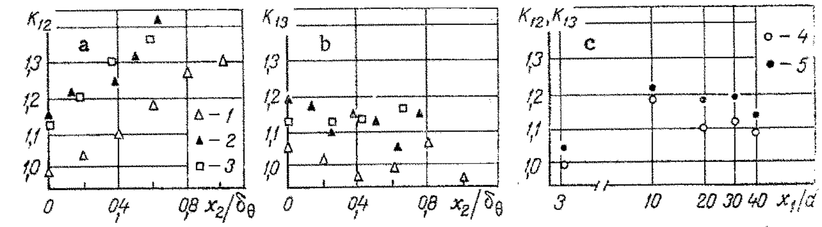

Fig. 2. Change in the coefficients K_{12} and K_{13} over the radius (a, b) and along the axis (c) of the wake: 1) x_1/d = 3; 2) 10; 3) 20; 4) K_{12}; 5) K_{13}.

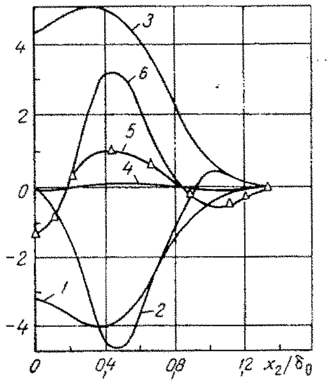

Fig. 3

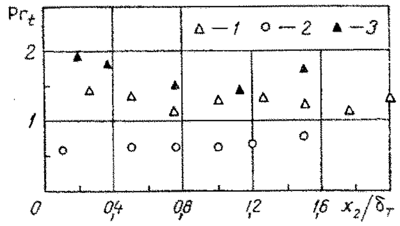

Fig. 4

Fig. 3. Balance of the mean square of the temperature fluctuations for for the section x_1/d = 50: 1) convection; 2) generation; 3) dissipation; 4) molecular diffusion; 5) turbulent diffusion (experiment); 6) turbulent diffusion (from balance equation (1)).

Fig. 4. Change in the turbulent Prandtl number over the radius of the wake: 1) [2]; 2) [7]; 3) our data.

The relationship between dissipation and turbulent diffusion is roughly the same for the flows being compared. The only term which exhibits a similar change and magnitude in the wakes of an ellipsoid and sphere is molecular diffusion. Thus, the completed analysis shows that physical processes involving the transport of temperature fluctuations in a wake behind bodies in a flow are governed to a significant extent by the shape of the body.

As was already noted in [1], the thermal wake has a longer "memory" in regard to the initial conditions. It is natural to expect this to be manifest in the distribution of the terms of balance equation (1). Taking into consideration the method used to introduce heat into the wake in the present experiment, it is interesting to compare out data on the balance of $\overline{\theta^2}$ with the results obtained for a circular coflowing stream [7]. Without discussing the details of numerical evaluation of the individual terms of Eq. (1), we can state that the distributions of convection, turbulent diffusion, generation, and dissipation are fully similar for the cases examined. This can be attributed to the existence of a jet section in the initial stage of development of the wake in the present experiment. Unfortunately, due to absence of the necessary data for the wake of a heated ellipsoid, it is not possible to evaluate the effect of the method used to create the thermal wake on the character of the distributions of the terms in Eq. (1).

The measured statistical characteristics of the velocity and temperature fields made it possible to determine the turbulent Prandtl number, which was calculated from the experimental profiles by means of the relation

$$\mathrm{Pr}_t = \frac{\overline{u_1 u_2}/\Delta U_0^2}{\overline{u_2\theta}/\Delta U_0 \Delta T_0} \frac{g}{f}. \tag{17}$$

The results in Fig. 4 show an approximate constancy of the parameter Pr_t over most of the radius of the wake, which is consistent with the data in [2]. The values of Pr_t calculated for a circular coflowing stream [7] are roughly half as great as the values we obtained. This can serve as confirmation of the nonuniversality of the turbulent Prandtl number for different types of flows which has been noted in several studies.

NOTATION

U, V, longitudinal and transverse components of velocity; $\Delta U = U - U_\infty$, mean-velocity defect; U_ℓ, longitudinal component of velocity on the wake axis at a distance of 0.1d from the model; T, mean temperature; $\Delta T = T - T_\infty$, excess temperature; $\Delta T_i = T_0 - T_\infty$, initial difference in temperatures on the wake axis at a distance of 0.1d from the model; d, maximum diameter of the ellipsoid; u_1, u_2, longitudinal and transverse fluctuations of velocity; θ, temperature fluctuations; x_1, x_2, x_3, longitudinal and transverse coordinates; δ_T, δ_θ, characteristic transverse dimensions of the wake, determined by the half-width of the profiles of T and θ; κ, diffusivity; C_T, C_U, C_δ, coefficients in the degeneration laws. Indices: bar, averaging over time; 0, parameters on the wake axis; ∞, parameters in the incoming flow.

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